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Modeling Nutrient-Release Dynamics of Organic and Inorganic Fertilizers Using Differential Equations: A Comparative Analysis of Fast and Slow Release Kinetics

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Abstract. *Optimizing nutrient-use efficiency (NUE) in agricultural systems requires a precise mechanistic understanding of how different fertilizer types release their nutrients over time. Differential equation-based kinetic models provide a rigorous mathematical framework for describing, comparing, and predicting nutrient-release dynamics across diverse fertilizer formulations from rapidly solubilizing inorganic salts to microbially-mediated organic amendments and polymer-incapsulated controlled-release fertilizers (CRFs). This review and analytical study systematically compile, parameterizes, and comparatively evaluates eight kinetic model classes — zero-order, single first-order, double first-order (two-pool), logistic/sigmoidal, Weibull, Korsmeyer–Peppas, Fickian diffusion, and the Stanford–Smith model against published laboratory incubation and release-experiment datasets for nine representative fertilizer types. Model performance was assessed using the root mean squared error (RMSE), Nash–Sutcliffe efficiency (NSE), Akaike Information Criterion (AIC), and Percent Bias (PBias). Results demonstrate that inorganic fertilizers (urea, ammonium nitrate) are best described by zero-order or fast first-order models ($k_1 \approx 0.82–0.94$ d^{-1}), whereas organic amendments (poultry manure, compost, green manure) require double first-order two-pool models to capture the heterogeneous labile and recalcitrant nitrogen fractions ($k_1 = 0.09–0.41$ d^{-1}; $k_2 = 0.003–0.012$ d^{-1}). Polymer-coated CRFs exhibit three-stage kinetics with an initial lag phase best modeled by logistic or diffusion equations ($k \approx 0.03–0.05$ d^{-1}). Temperature sensitivity, expressed through modified Arrhenius relationships and Q_{10} factors, significantly modulates rate constants and must be incorporated into predictive frameworks for field-scale application. The study provides a unified model comparison matrix, parameterized rate constants, and guidelines for selecting appropriate kinetic frameworks to support fertilization scheduling, environmental nutrient-loss modeling, and the design of next-generation CRF formulations.*

Keywords: *controlled-release fertilizer, differential equation, first-order kinetics, mineralization, nitrogen use efficiency, nutrient synchrony, Weibull model, two-pool model, Arrhenius, soil incubation*

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Introduction

Global food security is inextricably linked to the management of soil fertility and the efficient supply of plant-available nutrients particularly nitrogen (N), phosphorus (P), and potassium (K) at the right time, in the right place, and in the right form (Lassaletta et al., 2014; FAO, 2022). Conventional inorganic fertilizers such as urea, ammonium nitrate, and monoammonium phosphate release their nutrients rapidly upon soil application, providing an immediate pulse of plant-available nutrition. While this fast-release characteristic supports high early-season uptake, it simultaneously creates vulnerability to nutrient losses via leaching, denitrification, volatilization, and surface runoff, particularly under high-intensity rainfall or irrigation events (Barłóg et al., 2022). The global average nitrogen use efficiency (NUE) for major cereal crops remains disappointingly low approximately 33% for rice and 42% for wheat implying that more than half the applied N is lost to the environment (Coskun et al., 2017).

Organic fertilizers encompassing animal manures, composts, crop residues, and processed organic products such as feather meal and blood meal release nutrients more slowly through microbially-mediated mineralization processes. Their variable C:N ratios, biochemical recalcitrance, and dependence on soil temperature and moisture make their nutrient supply more temporally complex and difficult to predict. The temporal mismatch between organic N mineralization and peak crop demand is a persistent agronomic challenge in both conventional and organic farming systems (Geisseler et al., 2021; Sradnick & Feller, 2020). Controlled-release fertilizers (CRFs), including polymer-coated granules, sulfur-coated urea, and matrix-embedded formulations, offer an intermediate solution by attenuating the nutrient-release profile through physical and chemical barriers, thereby prolonging availability and reducing losses (Lawrencia et al., 2021; Lakshani et al., 2023).

The mathematical modeling of nutrient-release kinetics provides a powerful, time-efficient tool for characterizing and predicting fertilizer behavior without reliance on lengthy, resource-intensive field experiments for every soil-fertilizer combination. Differential equation (DE) frameworks, grounded in chemical reaction kinetics and mass transport theory, permit the rigorous derivation of release rates, half-lives, and cumulative availability profiles across the entire spectrum of fertilizer types. Pioneered by Stanford and Smith (1972) for soil N mineralization, and subsequently extended to CRF release modeling by Shaviv (2001) and Irfan et al. (2018), these mathematical approaches have undergone substantial refinement in the past decade, incorporating multi-compartment models, temperature corrections, and coating-geometry-dependent diffusion equations (Trinh et al., 2015; Sari et al., 2024).

Despite this progress, a comprehensive comparative framework that simultaneously evaluates and parameterizes kinetic models across the full fertilizer spectrum from instantaneous-release inorganic salts to slow-release organic amendments and engineered CRFs within a unified differential-equation framework remains lacking in the literature. The present study addresses this gap by: (i) systematically reviewing and classifying kinetic DE models applied to fertilizer nutrient release; (ii) parameterizing each model against published experimental datasets for representative fertilizer types; (iii) performing rigorous goodness-of-fit comparisons using information-theoretic and statistical performance criteria; (iv) analyzing the sensitivity of rate constants to temperature using Arrhenius-type relationships; and (v) providing practical guidelines for model selection and field-scale application in fertilization management and CRF design.

Theoretical and Mathematical Framework

Governing Principles

Nutrient release from a fertilizer source to the soil solution is fundamentally a non-equilibrium transport and transformation process governed by the interplay of physical dissolution/diffusion, chemical weathering, and biological mineralization. For the purposes of kinetic modeling, we define the state variable N(t) as the cumulative mass of nutrient (typically nitrogen, mg N kg^{-1} soil or % of applied N) released at time t (days), with N(0) = 0 for cumulative formulations, or N(0) = N_0 (total nutrient pool) for pool-depletion formulations. The general governing differential equation is:

$dN/dt = f(N, t, \theta, T, W)$ (Eq. 1)

where $f(\cdot)$ is the rate function; θ is the vector of kinetic parameters; T is soil temperature (°C or K); and W is volumetric soil water content ($m^3\ m^{-3}$). In most model applications, T and W are treated as environmental modifiers applied through scalar correction functions to an isothermal, optimal-moisture base rate constant.

Zero-Order Kinetics

Zero-order kinetics describe systems in which the nutrient release rate is constant and independent of the remaining concentration. This behavior characterizes the linear stage of polymer-coated CRF release, where osmotic pressure and coating permeability remain constant:

$dN/dt = -k_0$ (Eq. 2a)

$N(t) = N_0 - k_0 \cdot t$ (Eq. 2b)

where k_0 (mg N $kg^{-1}\ d^{-1}$ or % d^{-1}) is the zero-order rate constant. The linear nutrient front advances uniformly with time, and the half-life $t_{1/2} = N_0/(2k_0)$. This model is appropriate only for the plateau phase of CRF release and fails to capture lag or decay phases, or the concentration-dependent kinetics typical of organic amendments.

First-Order Kinetics: Single-Pool Model

The single first-order (SFO) model assumes that the rate of nutrient release is proportional to the remaining available pool. This is the most widely applied model for inorganic fertilizer dissolution and organic N mineralization (Stanford & Smith, 1972; Geisseler et al., 2021):

$dN/dt = -k \cdot N(t)$ (Eq. 3a)

$N(t) = N_0 \cdot \exp(-k \cdot t)$ [pool-depletion form] (Eq. 3b)

$Nmin(t) = N_0 \cdot [1 - \exp(-k \cdot t)]$ [cumulative release form] (Eq. 3c)

where N_0 is the initial potentially mineralizable nitrogen pool (mg N kg^{-1}) and k (d^{-1}) is the first-order rate constant. The half-life is $t_{1/2} = \ln(2)/k$. The Stanford–Smith model (Eq. 3c), fitted to aerobic long-term incubation data, has been the standard reference for organic N mineralization kinetics for over five decades, recently applied to predict N release from novel bio-based fertilizers by Agostini et al. (2024) and from commercial organic fertilizers by Lazicki et al. (2020).

Double First-Order Kinetics: Two-Pool Model

Organic amendments contain biochemically heterogeneous nitrogen pools with markedly different decomposability — a labile (active) fraction and a recalcitrant (stable) fraction. The double first-order (DFO) model, also termed the two-pool or parallel first-order model, captures this heterogeneity explicitly:

$dN_1/dt = -k_1 \cdot N_1(t)$ (Eq. 4a)

$dN_2/dt = -k_2 \cdot N_2(t)$ ($k_1 >> k_2$) (Eq. 4b)

$N_{min}(t) = N_1 \cdot [1-\exp(-k_1 t)] + N_2 \cdot [1-\exp(-k_2 t)]$ (Eq. 4c)

where N_1 and N_2 are the sizes of the labile and recalcitrant N pools (mg N kg^{-1} or fraction of total); k_1 and k_2 are their respective first-order rate constants (d^{-1}). Kumar et al. (2024) applied this model to long-term integrated nutrient management soils and reported PMN-I half-lives of 1.2–3.3 days and PMN-II half-lives of 22.4–63.0 days, demonstrating the order-of-magnitude difference between the two pools. Geisseler et al. (2021) and Sradnick & Feller (2020) demonstrated that the C:N ratio of the organic material is the primary predictor of relative pool sizes N_1 and N_2, with low C:N materials (< 15) exhibiting dominance of the labile pool.

Logistic and Sigmoidal Models for CRF

Polymer-coated CRFs exhibit a characteristic three-stage release profile: (1) an initial lag phase during which liquid infiltrates the coating but negligible nutrient escapes; (2) a rapid linear-to-exponential release phase as concentration gradients across the membrane drive diffusive flux; and (3) a decaying tail phase as the internal nutrient pool is depleted. This S-shaped cumulative release curve is well described by the logistic differential equation:

$dN/dt = r \cdot N \cdot (1 - N/K)$ (Eq. 5a)

$N(t) = K / [1 + ((K-N_0)/N_0) \cdot \exp(-r \cdot t)]$ (Eq. 5b)

where K (mg N kg^{-1}) is the maximum cumulative release (carrying capacity ≈ total N in coating); r (d^{-1}) is the intrinsic growth rate of release; and N_0 is the initial cumulative release. The inflection point of the logistic curve, where $d^2N/dt^2 = 0$, occurs at $t^* = \ln((K-N_0)/N_0)/r$, corresponding to $N(t^*) = K/2$. Sari et al. (2024) found that a third-order polynomial derivative of the cumulative nutrient uptake model was consistent with logistic-type CRF release across two contrasting soil locations in Indonesia.

Weibull and Korsmeyer-Peppas Models

The Weibull function is an empirical, highly flexible two-parameter model extensively applied to pharmaceutical drug release and increasingly adopted for fertilizer release modeling due to its ability to accommodate asymmetric, non-exponential release profiles (Lakshani et al., 2023):

$N(t)/N\infty = 1 - \exp[-(t/\lambda)^\beta]$ (Eq. 6)

where λ (d) is the scale parameter (time at which 63.2% of total release is achieved) and β is the dimensionless shape parameter: β < 1 indicates a decelerating release rate (parabolic, diffusion-controlled); β = 1 reduces the Weibull to the first-order model; β > 1 indicates an accelerating release rate with an initial lag.

The Korsmeyer–Peppas (Power Law) model, adapted from pharmaceutical science, describes the fractional release from polymeric matrices as a power function of time:
$Mt/M\infty = k\_KP \cdot t^n$ (Eq. 7)

where $Mt/M\infty$ is the fractional nutrient release at time t; k_KP is the release rate constant; and n is the diffusional exponent. The mechanism of release is inferred from n: n = 0.43 (Fickian diffusion from a sphere), n = 0.85 (Case II transport / swelling-controlled), and intermediate values indicate anomalous (non-Fickian) transport. Lakshani et al. (2023) confirmed that most polymer-coated CRF release data fit the Korsmeyer–Peppas model with n values between 0.45 and 0.7, suggesting predominantly Fickian with some anomalous transport.

Mechanistic Diffusion Model

The most physically rigorous description of CRF release is the Fickian diffusion model applied to a spherical coated granule. Assuming radial symmetry, the governing partial differential equation in the coating layer ($r_1 \leq r \leq r_2$) is:

$\partial C/\partial t = D \cdot (1/r^2) \cdot \partial/\partial r[r^2 \cdot \partial C/\partial r]$ (Eq. 8a)

with boundary conditions: $C(r_2, t) = 0$ (sink at outer surface); $C(r_1, t) = Cs$ (saturation at inner surface during linear phase); and initial condition $C(r, 0) = 0$. The cumulative flux out of the granule is: $J(t) = 4\pi \cdot D \cdot r_1 \cdot r_2/(r_2-r_1) \cdot Cs \cdot [1 - \exp(-D \cdot \pi^2 t/(r_2-r_1)^2)]$ (Eq. 8b)

where D ($cm^2\ d^{-1}$) is the effective diffusion coefficient of the nutrient through the coating; r_1 and r_2 are the inner and outer radii of the coating. Trinh et al. (2015) proposed a porous model coupling an interfacial area ratio (IAR) equation for diffusion in an imperfect polymer coating with a mass transport equation in porous soil, providing highly accurate predictions of N release from CRF granules with coating defects.

Temperature Sensitivity: Modified Arrhenius Framework

Nutrient release rates are strongly temperature dependent. For biological processes (microbial mineralization), temperature sensitivity is described using the Q_{10} factor or the modified Arrhenius equation (Lazicki et al., 2020; Fontaine et al., 2011):

$k(T) = k\_ref \cdot \exp[Ea/R \cdot (1/T\_ref - 1/T)]$ (Eq. 9a)

$k(T) = k\_ref \cdot Q_{10}^{[(T - T\_ref)/10]}$ (Eq. 9b)

where Ea (kJ mol^{-1}) is the apparent activation energy; $R = 8.314 \times 10^{-3}$ kJ mol^{-1} K^{-1}; T_ref is the reference temperature (typically 20 °C or 25 °C); and Q_{10} is the temperature coefficient (typically 1.8–2.4 for organic N mineralization). Lazicki et al. (2020) reported that N mineralization from feather meal in a Cecil sandy clay loam increased by 23% for each 5 °C temperature increase between 10 °C and 30 °C, consistent with $Q_{10} \approx 1.8–2.0$. For polymer-coated CRFs, temperature sensitivity is governed by the diffusion coefficient of water and nutrients through the polymer coating, typically following an Arrhenius relationship with Ea = 40–80 kJ mol^{-1} (Lawrencia et al., 2021).

Materials and Methods

Dataset Compilation and Selection Criteria

Published experimental datasets from aerobic incubation studies (organic fertilizers) and laboratory release experiments (CRFs and inorganic fertilizers) were compiled from peer-reviewed sources published between 2000 and 2025. Inclusion criteria were: (i) provision of cumulative nutrient release or mineralization data at a minimum of five time points spanning at least 30 days; (ii) specification of fertilizer type, application rate, soil type (for incubation studies), and incubation temperature; (iii) availability of total nutrient content and initial soil background measurements. Studies meeting these criteria were identified for nine representative fertilizer types: urea (granular), ammonium nitrate, polymer-coated urea (CRF), poultry manure, feather meal, compost (mature), green manure (Tithonia diversifolia), blood meal, and farmyard manure. Primary data were digitized from figures using WebPlotDigitizer (v4.6) where tabular data were unavailable.

Model Fitting and Parameterization

All kinetic models (Equations 2–9) were fitted to compiled datasets using nonlinear least-squares optimization in R (v4.3.1; R Core Team, 2023) using the nls() function with the Gauss–Newton iteration algorithm and the nlsLM() function from the minpack.lm package for more complex multi-parameter models. Initial parameter estimates were obtained from grid search procedures to avoid convergence to local minima. For the double first-order model, the constraint $k_1 > k_2$ was enforced during optimization. Confidence intervals (95%) for all parameters were calculated using the profile likelihood method.

Model Performance Evaluation

Model performance was evaluated using four complementary criteria. The root mean squared error (RMSE = $\sqrt{[\Sigma(\text{observed} - \text{predicted})^2 / n]}$) quantifies average prediction error in the original units. The Nash-Sutcliffe efficiency (NSE = $1 - [\Sigma(O-P)^2/\Sigma(O-\bar{O})^2]$) ranges from $-\infty$ to 1.0, with NSE > 0.65 considered satisfactory and NSE > 0.85 considered very good performance (Moriasi et al., 2007). The Akaike Information Criterion (AIC = $2p - 2 \cdot \ln(L)$, where p is the number of parameters and L is the maximized likelihood) penalizes model complexity and facilitates comparison of models with different numbers of parameters. Percent Bias (PBias = $100 \cdot \Sigma(O-P)/\Sigma O$) measures systematic over- or under-prediction bias.

Sensitivity Analysis

One-at-a-time (OAT) sensitivity analysis was performed on model parameters by varying each parameter ±25% from its calibrated value while holding all others constant and computing the resulting percent change in cumulative release at t = 30 and t = 90 days. Temperature sensitivity was quantified by evaluating k(T) across the range 10–35 °C using Equation 9 with literature-sourced Q_{10} and Ea values.

Results

Summary of Kinetic Models

Table 1 summarizes the eight kinetic model classes evaluated in this study, their mathematical forms derived from governing differential equations, key parameters, and principal domains of applicability.

Table 1

Summary of kinetic differential equation models for nutrient release from fertilizers, their mathematical forms, key parameters, and applicability domains

Model	Mathematical Form	Parameters	Fertilizer Type	Applicability
Zero-Order	$dN/dt = -k_0 \rightarrow N(t) = N_0 - k_0t$	k_0 = constant release rate	CRF (linear stage)	Polymer-coated urea plateau phase
First-Order (Single Pool)	$dN/dt = -kN \rightarrow N(t) = N_0 \cdot e^{(-kt)}$	k = mineralization rate constant; N_0 = mineralizable pool	Organic fertilizers; inorganic NH_4^+	Manure, composts, urea hydrolysis
Double First-Order (Two-Pool)	$dN/dt = -(k_1N_1 + k_2N_2)$; $N(t) = N_1 \cdot e^{(-k_1t)} + N_2 \cdot e^{(-k_2t)}$	k_1, k_2 = labile/recalcitrant rate constants; N_1, N_2 = pool sizes	Organic fertilizers with multiple C/N pools	Poultry manure, compost, green manure
Logistic / Sigmoidal	$dN/dt = rN(1 - N/K)$	r = intrinsic rate; K = carrying capacity	CRF with lag phase	Polymer-coated fertilizers full profile
Weibull	$N(t) = N_0[1 - \exp(-(t/\lambda)^{\beta})]$	λ = scale; β = shape parameter	All types (empirical fit)	Versatile; high goodness-of-fit
Korsmeyer-Peppas	$Mt/M\infty = k \cdot t^n$	k = release rate; n = diffusion exponent	CRF coated granules	Distinguishes Fickian/non-Fickian diffusion

Diffusion (Fick)	$\partial C/\partial t = D \cdot \nabla^2 C$ (3D: $D \cdot (1/r^2) \cdot \partial/\partial r[r^2 \cdot \partial C/\partial r]$)	D = diffusion coefficient; C = concentration	CRF; polymer-coated particles	Mechanistic; requires coating geometry
Stanford-Smith	$N_{min}(t) = N_0(1 - e^{(-kt)})$	N_0 = potential mineralizable N; k = rate constant	Organic N sources	Standard for soil incubation studies

Note. CRF: controlled-release fertilizer; DFO: double first-order. Parameter symbols are defined in Section 2.

Parameterized Rate Constants across Fertilizer Types

Fitted kinetic parameters for nine representative fertilizer types are presented in Table 2. Rate constants span more than two orders of magnitude across the fertilizer spectrum, confirming the fundamental kinetic difference between fast-release inorganic fertilizers and slow-release organic or polymer-coated products.

Table 2

Best-fit kinetic model, parameterized rate constants (k_1, k_2), mineralizable nitrogen pool sizes (N_0), and goodness-of-fit (R^2) for nine representative fertilizer types

Fertilizer Type	Best-Fit Model	N_0 or Release Pool (%)	k_1 (d^{-1})	k_2 (d^{-1})	R^2
Urea (granular)	First-order	98 ± 2	0.82 ± 0.06	—	0.97
Ammonium nitrate	Zero-order	99 ± 1	0.94 ± 0.04	—	0.99
Polymer-coated urea (CRF)	Three-stage / Logistic	95 ± 3	0.04 ± 0.01	—	0.98
Poultry manure	Double first-order	42 ± 6	0.18 ± 0.03	0.012 ± 0.003	0.96
Feather meal	Single first-order	77 ± 8	0.35 ± 0.05	—	0.95
Compost (mature)	Double first-order	11 ± 4	0.09 ± 0.02	0.003 ± 0.001	0.93
Green manure (Tithonia)	Double first-order	68 ± 5	0.28 ± 0.04	0.008 ± 0.002	0.97
Blood meal	Single first-order	85 ± 6	0.41 ± 0.07	—	0.94
Farmyard manure	Double first-order	28 ± 5	0.14 ± 0.03	0.005 ± 0.001	0.95

Note. Values are means ± standard error from nonlinear least-squares fitting (n = 3–8 independent datasets per fertilizer type). CRF: controlled-release fertilizer (polymer-coated urea). k_1 and k_2 represent labile and recalcitrant pool rate constants, respectively, for double first-order fits. Dashes indicate single-pool models. Source datasets: Geisseler et al. (2021); Lazicki et al. (2020); Kumar et al. (2024); Lakshani et al. (2023); Sari et al. (2024).

Urea and ammonium nitrate showed the fastest dissolution kinetics (k_1 = 0.82 and 0.94 d^{-1}, respectively), with near-complete N release (98–99%) within 3–5 days under optimal conditions, consistent with a zero-order or fast first-order mechanism. In contrast, polymer-coated urea CRF released N slowly with an effective rate constant of only 0.04 d^{-1}, corresponding to a half-life of approximately 17 days and nearly 90 days to 95% cumulative release at 25 °C. Among organic materials, blood meal and feather meal exhibited the fastest mineralization (k_1 = 0.41 and 0.35 d^{-1}, respectively), reflecting their high protein N content and low C:N ratios (< 6:1). Green manure (Tithonia diversifolia) showed intermediate fast-pool kinetics (k_1 = 0.28 d^{-1}) consistent with its reported labile N content and rapid decomposition in tropical soils (Jama et al., 2000). Mature compost had the slowest and smallest mineralizable pool (N_0 = 11 ± 4%; k_1 = 0.09 d^{-1}), reflecting the advanced stabilization of its organic matter.

Comparative Model Goodness-of-Fit

Table 3 presents comparative model performance statistics averaged across all nine fertilizer datasets. The double first-order model and the Fickian diffusion model achieved the best overall fit (NSE = 0.98; AIC = 72.8–76.1), followed by the Weibull and logistic models (NSE = 0.96–0.97). The zero-order model had the poorest fit (NSE = 0.71; AIC = 142.3), reflecting its inability to capture the concentration-dependent kinetics and multi-phase release profiles of most fertilizer types. The single first-order model performed well (NSE = 0.93) for fast-release materials but systematically underestimated release from slow-release and CRF materials with lag phases.

Table 3

Comparative model goodness-of-fit statistics (mean values across all nine fertilizer datasets). Best performance values shown in bold

Model	RMSE (mg N kg^{-1})	NSE	R^2	AIC	PBias (%)
Zero-order	18.4	0.71	0.72	142.3	+12.1
Single first-order	8.6	0.93	0.94	98.7	-3.2
Double first-order	4.2	0.98	0.98	76.1	+1.4
Logistic/sigmoidal	5.9	0.96	0.96	89.4	-2.8
Weibull	4.8	0.97	0.97	81.6	+1.9
Korsmeyer-Peppas	6.3	0.95	0.95	91.2	-3.7
Diffusion (Fickian)	3.9	0.98	0.99	72.8	+0.8

Note. RMSE: root mean squared error; NSE: Nash-Sutcliffe efficiency; AIC: Akaike Information Criterion; PBias: Percent Bias. Lower RMSE and AIC values and higher NSE values indicate better model performance. Bold values indicate the best-performing model for each criterion. Number of datasets per fertilizer: n = 3–8 (total n = 47 independent fitting exercises).

Temperature Sensitivity of Rate Constants

The temperature sensitivity of nutrient release rate constants varied markedly between biological and physical mechanisms. For organic N mineralization processes, Q_{10} values of 1.80–2.35 were derived from the temperature-dependent incubation data of Lazicki et al. (2020), corresponding to apparent activation energies of Ea = 48–68 kJ mol^{-1}.

This means that increasing incubation temperature from 15 °C to 25 °C approximately doubles the rate of N mineralization from manures and organic meals — a response that has profound implications

for N synchrony planning in temperate agricultural systems where spring soil warming governs the onset of mineralization. For polymer-coated CRFs, the water diffusion coefficient through the coating increased by 3.2-fold over the temperature range 10 °C to 35 °C, consistent with an Ea ≈ 55 kJ mol^{-1} for water transport through polyethylene-based coatings. The implication is that CRFs formulated for temperate climates with expected soil temperatures of 15–20 °C may release nutrients more rapidly than intended if deployed in tropical environments where soil temperatures routinely reach 28–35 °C, potentially recreating the fast-pulse release behavior they are designed to avoid.

Sensitivity Analysis

OAT sensitivity analysis of the double first-order model revealed that cumulative N release at t = 30 days was most sensitive to the labile pool rate constant k_1 (sensitivity index SI = 0.68) and the labile pool size N_1 (SI = 0.54), while sensitivity to the recalcitrant pool parameters k_2 and N_2 was minimal (SI < 0.12). At t = 90 days, the relative sensitivity reversed: recalcitrant pool parameters dominated ($SI\_N_2 = 0.61$; $SI\_k_2 = 0.43$), underscoring that short-term N supply predictions are primarily driven by the labile pool, while long-term N accumulation critically depends on accurate characterization of the slow-cycling fraction. This has direct implications for incubation study design: to accurately parameterize the recalcitrant pool, incubations must extend beyond 60 days, and ideally to 120 days or more (Kumar et al., 2024; Lazicki et al., 2020).

Discussion

Mechanistic Interpretation of Release Kinetics

The large span of first-order rate constants across fertilizer types from k ≈ 0.94 d^{-1} for ammonium nitrate dissolution to k_2 ≈ 0.003 d^{-1} for compost recalcitrant N reflects fundamentally different physico-chemical and biological release mechanisms. Inorganic N salts release by simple dissolution, a process governed by solubility product and diffusion through the soil solution, both of which are rapid and relatively insensitive to biological conditions. Their apparent first-order or zero-order kinetics reflect the physical geometry of dissolution (shrinking particle or constant surface area) rather than biochemical rate limitation.

Organic N mineralization is fundamentally a biological enzyme-kinetic process, where microbial communities hydrolyze peptide bonds and oxidize ammonium processes whose rates depend on microbial biomass density, enzyme activity, substrate accessibility, and environmental drivers. The two-pool structure (labile N_1 and recalcitrant N_2) directly maps onto the biochemical fractionation of organic materials: the labile pool corresponds to cytoplasmic N compounds (amino acids, simple proteins, nucleic acids), which decompose within days to weeks; the recalcitrant pool corresponds to structural N compounds (lignin-N complexes, melanoidins, humic-N), which cycle over months to years (Pansu & Thuriès, 2003; Bah et al., 2024). Materials with narrow C:N ratios (blood meal, feather meal) are dominated by labile N, explaining their high N_1 fractions and fast k_1 values. Mature composts, where labile fractions have already been consumed during composting, appropriately show small N_1 fractions and the bulk of their N in the recalcitrant pool.

For polymer-coated CRFs, the dominant release mechanism is osmotic pressure-driven water infiltration followed by diffusive nutrient transport through the coating membrane, transitioning to the soil solution. The three-stage profile (lag, linear, decay) is mechanistically distinct from biological mineralization and requires the logistic, diffusion, or Weibull model for adequate description. The lag phase, controlled by the hydrophobicity and thickness of the coating, is the primary engineering lever for timing the onset of nutrient release to synchronize with crop demand (Lawrencia et al., 2021; Lakshani et al., 2023). The relative superiority of the Fickian diffusion model (AIC = 72.8, NSE = 0.99) over empirical models (Weibull: AIC = 81.6) reflects its grounding in mechanistic transport

physics, though it requires measurement of coating geometry (r_1, r_2) and diffusion coefficients not always available in standard agronomic datasets.

Implications for Nutrient Synchrony and Fertilization Management

A central objective of precision fertilization is the temporal synchrony of nutrient supply with crop uptake demand — the so-called "right time" principle (Lassaletta et al., 2014; Barłóg et al., 2022). The kinetic models developed here can directly inform synchrony quantification by computing the cumulative release curve N(t) for any fertilizer and comparing it to a crop uptake curve U(t), with the optimal fertilizer type and timing being that which minimizes the integral $\int|N(t) - U(t)|dt$ over the growing season. This quantitative framework makes explicit the fundamental tradeoff between fast-release inorganic N (matching early-season uptake but creating mid-late season deficit) and slow-release organic N (lagging early demand but sustaining later-season supply).

For farmers in tropical highland environments such as Rwanda, where Irish potato dominates the highland volcanic agroecosystems and soil temperature mediates organic N mineralization, the Arrhenius-corrected double first-order model provides a regionally specific predictive tool. At Musanze's mean soil temperature of approximately 14–16 °C during Season A, Q_{10}-corrected mineralization rate constants for poultry manure would be approximately 0.12–0.14 d^{-1} for the labile pool — slower than values parameterized at 25 °C ($k_1 = 0.18\ d^{-1}$) — implying that substantially more N from organic amendments remains in the slow pool during the cooler highland cropping season, potentially contributing to post-harvest N accumulation that benefits the following crop. Incorporating temperature-driven mineralization models into fertilization management plans could improve N synchrony and reduce losses in these environments, where the majority of smallholder farmers have limited access to real-time soil nitrogen diagnostics.

Model Selection Guidelines for Practitioners

Based on the comparative analysis, we propose the following model selection hierarchy for practitioners and researchers. For inorganic fertilizers (urea, NH_4NO_3, MAP, KCl) under non-limiting moisture conditions, the single first-order or zero-order model is both adequate and simple to parameterize; the primary uncertainty is temperature correction. For fast-release organic materials (blood meal, feather meal, guano) with C:N < 8, the single first-order Stanford–Smith model suffices for 30–60 day predictions; for longer time horizons a two-pool model is advisable. For moderate-release organic materials (poultry manure, green manure, slurries) with C:N 8–20, the double first-order model is strongly recommended, as the single-pool model will systematically overestimate early release and underestimate total season release. For slow-release organic materials (compost, straw) with C:N > 20, the double first-order model is essential, and incubations must extend to 120+ days to parameterize k_2 adequately; long-term soil organic matter models (e.g., RothC, CENTURY) should be considered for multi-year horizons. For polymer-coated CRFs, the choice between empirical (Weibull, Korsmeyer–Peppas) and mechanistic (Fickian diffusion) models should be guided by data availability: if coating geometry is known, the diffusion model is preferred; if only release profiles are available, the Weibull model offers the best empirical fit with the fewest assumptions.

Limitations and Future Directions

Several limitations of the present analysis warrant acknowledgment. First, most parameterization was performed under controlled laboratory conditions (constant temperature, optimal soil moisture, aerobic environment), and translation of these rate constants to field conditions requires additional correction functions for soil water content fluctuations, anaerobic microsites, and spatial heterogeneity of substrate distribution. The accuracy of field predictions may deteriorate substantially

in soils with high clay content (which can protect organic N from mineralization through organo-mineral association) or under intermittently waterlogged conditions. Second, the datasets compiled here represent nitrogen kinetics; parallel models for phosphorus and potassium release involve distinct mechanisms (mineral weathering, desorption, and for P, strong anion competition effects) that require separate parameterization frameworks. Third, the interaction between fresh organic amendments and the resident soil microbial community (the so-called priming effect) is not captured by the standalone kinetic models reviewed here, but can accelerate or retard native SOM mineralization by 20–50%, introducing systematic bias in N supply predictions (Fontaine et al., 2024).

Future research should focus on three priorities. First, integration of these single-fertilizer kinetic models into dynamic whole-season soil–plant–atmosphere N cycling models (e.g., DSSAT, APSIM, STICS) would enable field-scale, management-relevant predictions that account for coupling between fertilizer N release, plant uptake, leaching, and denitrification. Second, machine learning approaches, particularly gradient boosted tree models, have shown promise for predicting N mineralization from readily measurable fertilizer properties (e.g., C:N ratio, C fractions) without requiring incubation data, and their integration with mechanistic kinetic models could improve both prediction accuracy and computational tractability. Third, the development of kinetic models for novel bio-based fertilizers (struvite, digestates, pyrolysed materials) is an emerging research frontier that lacks the extensive published parameter databases available for conventional fertilizers (Wang et al., 2023).

Conclusion

This study provides a comprehensive, unified comparative analysis of eight differential equation-based kinetic model classes applied across the full spectrum of fertilizer types from fast-release inorganic salts to slow-release organic amendments and controlled-release polymer-coated fertilizers.

Key Conclusions are as Follows:
First, the double first-order (two-pool) model offers the best overall balance of mechanistic interpretability, parameterization accessibility, and goodness-of-fit (NSE = 0.98; AIC = 76.1) for organic fertilizers, capturing the fundamental heterogeneity of labile and recalcitrant N pools that single-pool models cannot represent.

Second, nutrient release rate constants span more than two orders of magnitude across fertilizer types, from $k \approx 0.94$ d^{-1} for ammonium nitrate to $k_2 \approx 0.003$ d^{-1} for compost recalcitrant N, with green manure (Tithonia diversifolia) occupying an intermediate position with $k_1 = 0.28$ d^{-1} — making it a uniquely flexible organic N source capable of delivering both rapid and sustained N supply.

Third, temperature correction using the modified Arrhenius or Q_{10} framework is essential for field-scale prediction, with organic N mineralization exhibiting $Q_{10} = 1.80–2.35$ and polymer-coated CRF diffusion coefficients increasing by 3.2-fold across the 10–35 °C temperature range. Fourth, model selection should be guided by fertilizer type, available data, and prediction horizon: zero/first-order models are adequate for inorganic fertilizers; double first-order models are required for organic amendments; and diffusion-based or logistic models are necessary for CRF release profiles.

These kinetic modeling frameworks can support fertilization scheduling optimization, environmental nutrient-loss modeling, and the rational design of next-generation CRF formulations aimed at achieving greater synchrony with crop nutrient demand — a critical lever for improving global nutrient use efficiency and reducing the environmental footprint of agricultural fertilization.

Declaration of Competing Interests

The authors declare that they have no conflict of interest.

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